確率論 (*Probability Theory*)

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1　これまでの課題

第 2 週

別紙の通り。

第 3 週

別紙の通り。

第 4 週

配布済み。

第 5 週

参考書 3.1。
この問題では、

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy = x + \left[\frac{y^2}{2}\right]_0^1 = x + \frac{1}{2} \tag{1}$$

$$f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx = \left[\frac{x^2}{2}\right]_0^1 + y = \frac{1}{2} + y \tag{2}$$

$$f_X(x)\,f_Y(y) = \left(x + \frac{1}{2}\right)\left(\frac{1}{2} + y\right) \neq f_{X,Y}(x,y) \tag{3}$$

$$F_{X,Y}(x,y) = \int_{-\infty}^{x}\int_{-\infty}^{y} (u+v)\,dudv = \frac{1}{2}\left(x^2 y + y^2 x\right) \tag{4}$$

$$g_X(x) = \int_{-\infty}^{\infty} g_{X,Y}(x,y)\,dy = x + \frac{1}{2} \tag{5}$$

$$g_Y(y) = \int_{-\infty}^{\infty} g_{X,Y}(x,y)\,dx = \frac{1}{2} + y \tag{6}$$

$$g_X(x)\,g_Y(y) = \left(x + \frac{1}{2}\right)\left(\frac{1}{2} + y\right) = g_{X,Y}(x,y) \tag{7}$$

$$G_{X,Y}(x,y) = \int_{-\infty}^{x}\int_{-\infty}^{y} (u+v)\,dudv = \frac{1}{4}\left(x^2 y^2 + x^2 y + y^2 x + yx\right) \tag{8}$$

を先に求めておけば、順に答えが求まる。

和の分布だけ更に、

$$f_{X+Y}(u) = \int_0^u f_{X,Y}(v, u-v)\,dv = \int_0^u (v + u - v)\,dv \tag{9}$$

$$F_{X+Y}(u) = \int_0^u f_{X+Y}(v)\,dv \tag{10}$$

$$g_{X+Y}(u) = \int_0^u g_{X,Y}(v, u-v)\,dv = \int_0^u \left(v(u-v) + \frac{1}{2}v + \frac{1}{4}\right)dv \tag{11}$$

$$G_{X+Y}(u) = \int_0^u g_{X+Y}(v)\,dv \tag{12}$$

など計算しておく必要がる。

第 6 週

第 5 週と同じ。

第 7 週

参考書 3.3, 3.4。
3.4

$$\begin{aligned}
Cov(X_1, X_2) &= \mathrm{E}[(X_1 - E[X_1])(X_2 - E[X_2])] \\
&= \mathrm{E}[X_1X_2 - \mathrm{E}[X_1]X_2 - X_1\mathrm{E}[X_2] + \mathrm{E}[X_1]\mathrm{E}[X_2]] \\
&= \mathrm{E}[X_1X_2] - 2\mathrm{E}[X_1]\mathrm{E}[X_2] + \mathrm{E}[X_1]\mathrm{E}[X_2] \\
&= \mathrm{E}[X_1X_2] - \mathrm{E}[X_1]\mathrm{E}[X_2]
\end{aligned} \tag{13}$$

$$\begin{aligned}
Cov(Y_1, Y_2) &= \mathrm{E}[(Y_1 - E[Y_1])(Y_2 - E[Y_2])] \\
&= \mathrm{E}[Y_1Y_2 - \mathrm{E}[Y_1]Y_2 - Y_1\mathrm{E}[Y_2] + \mathrm{E}[Y_1]\mathrm{E}[Y_2]] \\
&= \mathrm{E}[Y_1Y_2] - 2\mathrm{E}[Y_1]\mathrm{E}[Y_2] + \mathrm{E}[Y_1]\mathrm{E}[Y_2] \\
&= \mathrm{E}[Y_1Y_2] - \mathrm{E}[Y_1]\mathrm{E}[Y_2]
\end{aligned} \tag{14}$$

$$\begin{aligned}
\rho(Y_1, Y_2) &= \frac{Cov[Y_1, Y_2]}{\sqrt{Var[Y_1]\,Var[Y_2]}} \\
&= \frac{\mathrm{E}[Y_1Y_2] - \mathrm{E}[Y_1]\mathrm{E}[Y_2]}{a^2\sqrt{Var[X_1]\,b^2Var[X_2]}} \\
&= \frac{ab\mathrm{E}[X_1X_2] - \mathrm{E}[X_1]\mathrm{E}[X_2]}{a^2\sqrt{Var[X_1]\,b^2Var[X_2]}} \\
&= \frac{ab}{|ab|}\rho(X_1, X_2) \\
&= \pm\rho(X_1, X_2)
\end{aligned} \tag{15}$$